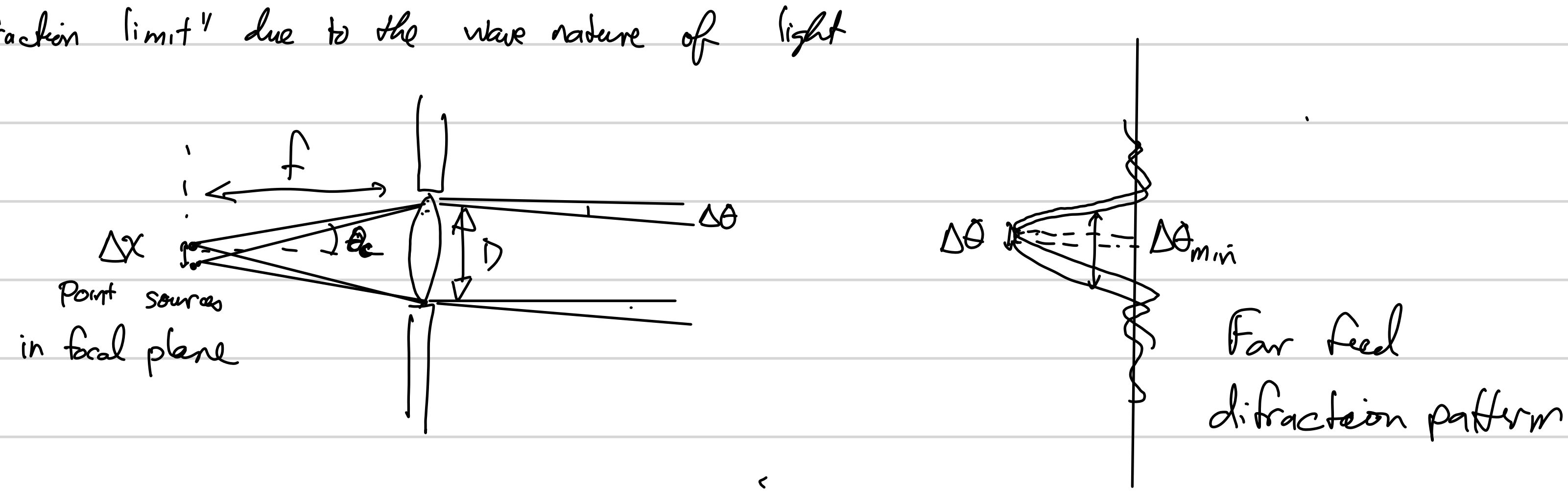
Physics 491: Lecture 5b Supplement

Heisenberg Microscope

The uncertainty principle has many implicitations. First and foremost, as we have seen, it is impossible to prepare a particle with a perfectly well-defined position and momentum. That is there is no wavefunction such that the probability of finding the particle in a certain interval $x \to x + \Delta x$ and simultaneously $p \to p + \Delta p$ is close to 1 for arbitrarilly small Δx and Δp. If we increase our certainty about the position Δx, we must reduce our certainty in the momentum Δp, and vice versa.

Heisenberg first presented his ideas in the context of the uncontrollable backaction of a quantum measurement, a fundamental a irreducible part of the quantum world. Any measurement involves a physical process. In performing a measurement, no matter how delicately, we will always disturb the system. This is generally true in the classical world as well, but in quantum mechanics theres no getting around it. This is, in some sense, due to the corpuscular quantized nature of particles a fields.

As a gedanken (thought) experiment, Heisenberg gave the first example - the "Heisenberg microscope." Any microscope has a certain "resolving power." That is is can only distinguish point separated by a minimal distance Δx. In a typical imaging system, this resolution is determined by the "diffraction limit" due to the wave nature of light



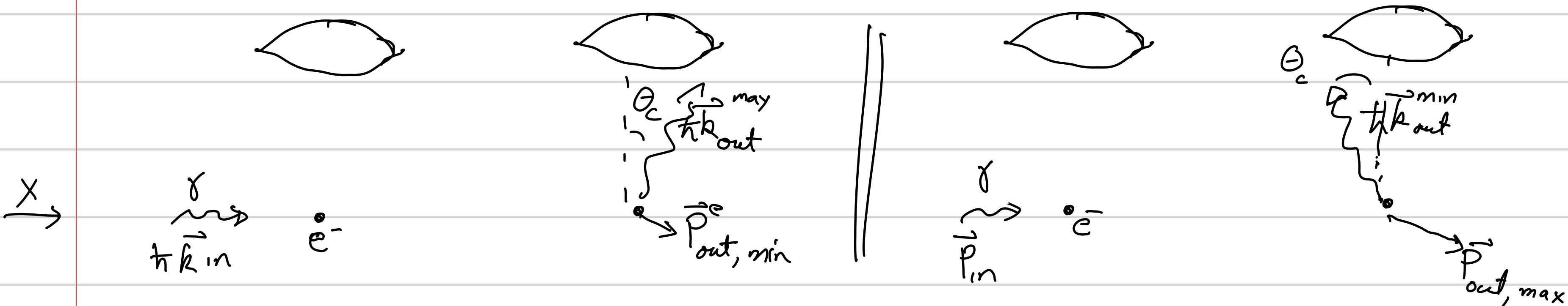
The minimum angular resolution $\Delta\theta_{min} \cong \frac{\lambda}{D}$, the angular width of the diffraction pattern of the point source through the aperature. Two points separated by Δx in the focal plane have an angular separation $\Delta\theta \cong \frac{\Delta x}{f}$

Thus the microscope has a resolution $\Delta x_{min} \cong \lambda \frac{f}{D} = \frac{\lambda}{2\theta_c}$

where θ_c is the "collection angle" ($2\theta_c$ is known as the "numercal aperature" in optics).

Now Heisenberg argued a follows. When we localize the particle (he considered an electron) the electron must interact with light (consisting of photons). Light is scattered into the microscope objective. And since these photons have an irreducible amount of momentum, some of this is transfered to the electron. But there are a range of different trajectories consistent with the imaging system, and because we don't know which one "happened", we are uncertain as to which one "happened." Thus we become uncertain about the momentum. Localizing the particle in position increases our uncertainty in momentum – uncontrollable measurement backaction.

Consider thus, different photon electron collision process, consistent with the collection angle θ_c



By conservation of momentum along the x-direction,

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\sin\theta_c + p^e_{x,min} \quad , \quad \frac{h}{\lambda} = -\frac{h}{\lambda''}\sin\theta_c + p^e_{x,max}$$

Assuming the collision is near elastic $\omega' \approx \omega'' \approx \omega \Rightarrow \lambda \approx \lambda' \approx \lambda''$

And, for small angles $\sin\theta_c \approx \theta_c \Rightarrow p^e_{x,max} - p^e_{x,min} \approx \frac{2h}{\lambda}\theta_c = \frac{h}{\Delta x_{min}}$

$$\Rightarrow \Delta p_x\Big|_{\text{back-action}} \cong \frac{h}{\Delta x_{min}}$$

Note, we have argued here "semiclassically." That is, we have use "classical probability" rather "probability amplitudes" to estimated to resolution/disturbance tradeoff. Nevertheless, the arguement contains the essential features. Measurements involve physical interactions, and the act of measuring disturbs the system being measured. What is different in the quantum world is that no matter what we do, there is no getting around it, no matter how hard we try. Measurement backaction is irreducible and fundamental.

We will see latter that the uncertainty principle applys not only to position and momentum, but other variables, like the components of angular momentum. Different physical quantity cannot be known simultaneously, and resolving one will necessarily disturb the complementary variable.

Finally, I want to dispell another myth propagated in the literature. Often one hears that one cannot measure a particle's position and momentum at the same time. Hogwash! We do it all the time. When we cross the street we see a car coming towards us. We estimate is position and momentum, and sometimes we jay walk! There's no violation of the uncertainty principle. It's just that the joint resolution of our measurement cannot violate $\Delta x \Delta p \geq \hbar/2$. Luckily $\hbar$ is tiny, so this constraint won't limit us enough to get run over!